# Observations on the Hyperbola $2x^2 - 5y^2 = 27$

D.Maheswari
Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Trichy-620002, India.

S.Dharuna
M.phil scholar, Department of Mathematics, Shrimati Indira Gandhi College, Trichy-620002, India.

**Abstract – The hyperbola represented by the binary quadratic equation $2x^2 - 5y^2 = 27$ is analyzed for finding its non-zero distinct integer solutions. A few interesting relations among its solutions are presented. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.**

**Index Terms – Binary quadratic, Hyperbola, Parabola, Pell equation, Integral solutions.**

## 1. INTRODUCTION

The hyperbola represented by the Diophantine equations of the form $ax^2 - by^2 = N, (a,b,c \neq 0)$ are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of $a, b$ and $N$. For an extensive review, one may refer [1-16].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation given by $2x^2 - 5y^2 = 27$ representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation, special Pythagorean triangle is constructed.

## 2. METHOD OF ANALYSIS

The binary quadratic equation representing hyperbola is given by

$$2x^2 - 5y^2 = 27 \tag{1}$$

Taking

$$\left.\begin{aligned} x &= X + 5T \\ y &= X + 2T \end{aligned}\right\} \tag{2}$$

in (1), it reduced to the equation

$$X^2 = 10T^2 - 9 \tag{3}$$

The smallest positive integer solution $(T_0, X_0)$ of (3) is

$$T_0 = 1, \quad X_0 = 1$$

To obtain the other solution of (3), consider the Pellian equation

$$X^2 = 10T^2 + 1 \tag{4}$$

whose smallest positive integer solution is

$$\tilde{T}_0 = 6, \quad \tilde{X}_0 = 19$$

The general solution $(\tilde{T}_n, \tilde{X}_n)$ of (4) is given by

$$\tilde{X}_n + \sqrt{10}\tilde{T}_n = \left(19 + 6\sqrt{10}\right)^{n+1}, n = 0,1,2,K \tag{5}$$

Since irrational roots occur in pair , we have

$$\tilde{X}_n - \sqrt{10}\tilde{T}_n = \left(19 + 6\sqrt{10}\right)^{n+1}, n = 0,1,2,K \tag{6}$$

From (5) and (6), solving for $\tilde{X}_n, \tilde{T}_n$ , we have

$$\tilde{X}_n = \frac{1}{2}\left[\left(19 + 6\sqrt{10}\right)^{n+1} + \left(19 - 6\sqrt{10}\right)^{n+1}\right] = \frac{1}{2} f_n$$

$$\tilde{T}_n = \frac{1}{2\sqrt{10}}\left[\left(19 + 6\sqrt{10}\right)^{n+1} - \left(19 - 6\sqrt{10}\right)^{n+1}\right] = \frac{1}{2\sqrt{10}} g_n$$

Applying Brahmagupta lemma between the solutions $(T_0, X_0)$ and $(\tilde{T}_n, \tilde{X}_n)$. The general solution $(T_{n+1}, X_{n+1})$ of (3) is found to be

$$T_{n+1} = X_0\tilde{T}_n + T_0\tilde{X}_n$$

$$X_{n+1} = X_0\tilde{X}_n + 10T_0\tilde{T}_n$$

$$\Rightarrow \quad T_{n+1} = \frac{1}{2\sqrt{10}} g_n + \frac{1}{2} f_n \tag{7}$$

$$X_{n+1}=\frac{1}{2}f_n+\frac{\sqrt{10}}{2}g_n \qquad (8)$$

Using (7) and (8) in (2), we have

$$x_{n+1}=X_{n+1}+5T_{n+1}=3f_n+\frac{15}{2\sqrt{10}}g_n \qquad (9)$$

$$y_{n+1}=X_{n+1}+2T_{n+1}=\frac{3}{2}f_n+\frac{6}{\sqrt{10}}g_n \qquad (10)$$

Thus (9) and (10) represent the integer solutions of hyperbola (1).

A few numerical examples are given in the following Table: 2.1 below:

Table: 2.1 NUMERICAL EXAMPLES

| $n$ | $x_{n+1}$ | $y_{n+1}$ |
|---|---|---|
| -1 | 6 | 3 |
| 0 | 204 | 129 |
| 1 | 7746 | 4899 |
| 2 | 294144 | 186033 |

Recurrence relations for $x$ and $y$ are

$$x_{n+3}-38x_{n+2}+x_{n+1}=0, n=-1,0,1\ldots$$
$$y_{n+3}-38y_{n+2}+y_{n+1}=0, n=-1,0,1\ldots$$

2.1 A few interesting relations among the solutions are given below

1. $19x_{n+1}-x_{n+2}+30y_{n+1}=0$
2. $x_{n+1}-19x_{n+2}+30y_{n+2}=0$
3. $19x_{n+1}-721x_{n+2}+30y_{n+3}=0$
4. $721x_{n+1}-x_{n+3}+1140y_{n+1}=0$
5. $x_{n+1}-x_{n+3}+60y_{n+2}=0$
6. $x_{n+1}-721x_{n+3}+1140y_{n+3}=0$
7. $12x_{n+1}+19y_{n+1}-y_{n+2}=0$
8. $x_{n+1}-19x_{n+2}+30y_{n+2}=0$
9. $12x_{n+1}+721y_{n+2}-19y_{n+3}=0$
10. $456x_{n+1}+721y_{n+1}-y_{n+3}=0$
11. $721x_{n+2}-19x_{n+3}+30y_{n+1}=0$
12. $19x_{n+2}-x_{n+3}+30y_{n+2}=0$
13. $x_{n+2}-19x_{n+3}+30y_{n+3}=0$
14. $12x_{n+2}+y_{n+1}-19y_{n+2}=0$
15. $24x_{n+2}+y_{n+1}-y_{n+3}=0$
16. $12x_{n+2}+19y_{n+2}-y_{n+3}=0$
17. $12x_{n+3}+19y_{n+1}-721y_{n+2}=0$
18. $456x_{n+3}+y_{n+1}-721y_{n+3}=0$
19. $12x_{n+3}+y_{n+2}-19y_{n+3}=0$

2.2 Each of the following expression represents a cubic integer

i. $\frac{1}{810}\left[1290x_{3n+3}-30x_{3n+4}+3870x_{n+1}-90x_{n+2}\right]$

ii. $\frac{1}{30780}\left[48990x_{3n+3}-30x_{3n+5}+146970x_{n+1}-90x_{n+3}\right]$

iii. $\frac{1}{27}\left[24x_{3n+3}-30y_{3n+3}+72x_{n+1}-90y_{n+1}\right]$

iv. $\frac{1}{513}\left[816x_{3n+3}-30y_{3n+4}+2448x_{n+1}-90y_{n+2}\right]$

v. $\frac{1}{19467}\left[30984x_{3n+3}-30y_{3n+5}+92952x_{n+1}-90y_{n+3}\right]$

vi. $\frac{1}{810}\left[48990x_{3n+4}-1290x_{3n+5}+146970x_{n+2}-3870x_{n+3}\right]$

vii. $\frac{1}{513}\left[24x_{3n+4}-1290y_{3n+3}+72x_{n+2}-3870y_{n+1}\right]$

viii. $\frac{1}{27}\left[816x_{3n+4}-1290y_{3n+4}+2448x_{n+2}-3870y_{n+2}\right]$

ix. $\frac{1}{513}\left[30984x_{3n+4}-1290y_{3n+5}+92952x_{n+2}-3870y_{n+3}\right]$

x. $\frac{1}{19467}\left[24x_{3n+5}-48990y_{3n+3}+72x_{n+3}-146970y_{n+1}\right]$

xi. $\frac{1}{513}\left[816x_{3n+5}-48990y_{3n+4}+2448x_{n+3}-146970y_{n+2}\right]$

xii. $\frac{1}{27}\left[30984x_{3n+5}-48990y_{3n+5}+92952x_{n+3}-146970y_{n+3}\right]$

xiii. $\frac{1}{324}\left[24y_{3n+4}-816y_{3n+3}+72y_{n+2}-2448y_{n+1}\right]$

xiv. $\frac{1}{12312}\left[24y_{3n+5}-30984y_{3n+3}+72y_{n+3}-92952y_{n+1}\right]$

xv. $\frac{1}{324}\left[816y_{3n+5}-30984y_{3n+4}+2448y_{n+3}-92952y_{n+2}\right]$

2.3 Each of the following expression represents a bi-quadratic integer

i. $\frac{1}{(810)^2}\left[\begin{array}{l}(1044900x_{4n+4}-24300x_{4n+5})\\+4(1290x_{n+1}-30x_{n+2})^2-1312200\end{array}\right]$

ii. $\frac{1}{(30780)^2}\left[\begin{array}{l}(1507912200x_{4n+4}-923500x_{4n+6})\\+4(48990x_{n+1}-30x_{n+3})^2-1894816800\end{array}\right]$

iii. $\frac{1}{(27)^2}\left[\begin{array}{l}(648x_{4n+4}-810y_{4n+4})\\+4(24x_{n+1}-30y_{n+1})^2-1458\end{array}\right]$

iv. $\frac{1}{(513)^2}\left[\begin{array}{l}(418608x_{4n+4}-15390y_{4n+5})\\+4(816x_{n+1}-30y_{n+2})^2-526338\end{array}\right]$

v. $\frac{1}{(19467)^2}\left[\begin{array}{l}(603165528x_{4n+4}-584010y_{4n+6})\\+4(30984x_{n+1}-30y_{n+3})^2-757928178\end{array}\right]$

vi. $\frac{1}{(19467)^2}\left[\begin{array}{l}(603165528x_{4n+4}-584010y_{4n+6})\\+4(30984x_{n+1}-30y_{n+3})^2-757928178\end{array}\right]$

vii. $\frac{1}{(513)^2}\left[\begin{array}{l}(12312x_{4n+5}-661770y_{4n+4})\\+4(24x_{n+2}-1290y_{n+1})^2-526338\end{array}\right]$

viii. $\frac{1}{(27)^2}\left[\begin{array}{l}(22032x_{4n+5}-34830y_{4n+5})\\+4(816x_{n+2}-1290y_{n+2})^2-1458\end{array}\right]$

ix. $\frac{1}{(513)^2}\left[\begin{array}{l}(15894792x_{4n+5}-661770y_{4n+6})\\+4(30984x_{n+2}-1290y_{n+3})^2-526338\end{array}\right]$

x. $\frac{1}{(19467)^2}\left[\begin{array}{l}(467208x_{4n+6}-953688330y_{4n+4})\\+4(24x_{n+3}-48990y_{n+1})^2-757928178\end{array}\right]$

xi. $\frac{1}{(513)^2}\left[\begin{array}{l}(418608x_{4n+6}-25131870y_{4n+5})\\+4(816x_{n+3}-48990y_{n+2})^2-526338\end{array}\right]$

xii. $\frac{1}{(27)^2}\left[\begin{array}{l}(836568x_{4n+6}-1322730y_{4n+6})\\+4(30984x_{n+3}-48990y_{n+3})^2-1458\end{array}\right]$

xiii. $\frac{1}{(324)^2}\left[\begin{array}{l}(7776y_{4n+5}-264384y_{4n+4})\\+4(24y_{n+2}-816y_{n+1})^2-209952\end{array}\right]$

xiv. $\frac{1}{(12312)^2}\left[\begin{array}{l}(295488y_{4n+6}-381475008y_{4n+4})\\+4(24y_{n+3}-30984y_{n+1})^2-303170688\end{array}\right]$

xv. $\frac{1}{(324)^2}\left[\begin{array}{l}(264384y_{4n+6}-10038816y_{4n+5})\\+4(816y_{n+3}-30984y_{n+2})^2-209952\end{array}\right]$

2.4 Each of the following expression represents a nasty number

i. $\frac{1}{810}\left[7740x_{2n+2}-180x_{2n+3}+9720\right]$

ii. $\frac{1}{30780}\left[293940x_{2n+2}-180x_{2n+4}+369360\right]$

iii. $\frac{1}{27}\left[144x_{2n+2}-180y_{2n+2}+324\right]$

iv. $\frac{1}{513}\left[4896x_{2n+2}-180y_{2n+3}+6156\right]$

v. $\frac{1}{19467}\left[185904x_{2n+2}-180y_{2n+4}+233604\right]$

vi. $\frac{1}{810}\left[293940x_{2n+3}-7740x_{2n+4}+9720\right]$

vii. $\frac{1}{513}\left[144x_{2n+3}-7740y_{2n+2}+6156\right]$

viii. $\frac{1}{27}\left[4896x_{2n+3}-7740y_{2n+3}+324\right]$

ix. $\frac{1}{513}\left[185904x_{2n+3}-7740y_{2n+4}+6156\right]$

x. $\frac{1}{19467}\left[144x_{2n+4}-293940y_{2n+2}+233604\right]$

xi. $\frac{1}{513}\left[4896x_{2n+4}-293940y_{2n+3}+6156\right]$

xii. $\frac{1}{27}\left[185904x_{2n+4}-293940y_{2n+4}+324\right]$

xiii. $\frac{1}{324}\left[144y_{2n+3}-4896y_{2n+2}+3888\right]$

xiv. $\frac{1}{12312}\left[144y_{2n+4}-185904y_{2n+2}+147744\right]$

xv. $\frac{1}{324}\left[4896y_{2n+4}-185904y_{2n+3}+3888\right]$

2.5 Each of the following expression represents a quintic integer

i. $\frac{1}{(810)^3}\begin{bmatrix}846369000x_{5n+5}-19683000x_{5n+6}\\+5(1290x_{n+1}-30x_{n+2})^3-4231845000x_{n+1}\\+98415000x_{n+2}\end{bmatrix}$

ii. $\frac{1}{(30780)^3}\begin{bmatrix}46413537516000x_{5n+5}\\-28422252000x_{5n+7}\\+5(48990x_{n+1}-30x_{n+3})^3\\-232067687580000x_{n+1}\\+142111260000x_{n+3}\end{bmatrix}$

iii. $\frac{1}{(27)^3}\begin{bmatrix}17496x_{5n+5}-21870y_{5n+5}+\\5(24x_{n+1}-30y_{n+1})^3-87480x_{n+1}\\+109350y_{n+1}\end{bmatrix}$

iv. $\frac{1}{(513)^3}\begin{bmatrix}214745904x_{5n+5}-7895070y_{5n+6}\\+5(816x_{n+1}-30y_{n+2})^3-1073729520x_{n+1}\\+39475350y_{n+2}\end{bmatrix}$

v. $\frac{1}{(19467)^3}\begin{bmatrix}11741823333576x_{5n+5}\\-11368922670y_{5n+7}\\+5(30984x_{n+1}-30y_{n+3})^3\\-58709116667880x_{n+1}\\-56844613350y_{n+3}\end{bmatrix}$

vi. $\frac{1}{(810)^3}\begin{bmatrix}32142339000x_{5n+6}-846369000x_{5n+7}\\5(48990x_{n+2}-1290x_{n+3})^3\\-160711695000x_{n+2}+4231845000x_{n+3}\end{bmatrix}$

vii. $\frac{1}{(513)^3}\begin{bmatrix}6316056x_{5n+6}-339488010y_{5n+5}\\+5(24x_{n+2}-1290y_{n+1})^3\\-31580280x_{n+2}+1697440050y_{n+1}\end{bmatrix}$

viii. $\frac{1}{(27)^3}\begin{bmatrix}594864x_{5n+6}-940410y_{5n+6}\\+5(816x_{n+2}-1290y_{n+2})^3\\-2974320x_{n+2}+4702050y_{n+2}\end{bmatrix}$

ix. $\frac{1}{(513)^3}\begin{bmatrix}8154028296x_{5n+6}-339488010y_{5n+7}\\+5(30984x_{n+2}-1290y_{n+3})^3\\-40770141480x_{n+2}+1697440050y_{n+3}\end{bmatrix}$

x. $\frac{1}{(19467)^3}\begin{bmatrix}9095138136x_{5n+7}-18565450720110y_{5n+5}\\+5(24x_{n+3}-48990y_{n+1})^3\\-45475690680x_{n+3}+92827253600550y_{n+1}\end{bmatrix}$

xi. $\frac{1}{(513)^3}\begin{bmatrix}214745904x_{5n+7}-12892649310y_{5n+6}\\+5(816x_{n+3}-48990y_{n+2})^3\\-1073729520x_{n+3}+64463246550y_{n+2}\end{bmatrix}$

xii. $\frac{1}{(27)^3}\begin{bmatrix}22587336x_{5n+7}-35713710y_{5n+7}\\+5(30984x_{n+3}-48990y_{n+3})^3\\-112936680x_{n+3}+178568550y_{n+3}\end{bmatrix}$

xiii. $\frac{1}{(324)^3}\begin{bmatrix}2519424y_{5n+6}-85660416y_{5n+5}\\+5(24y_{n+2}-816y_{n+1})^3\\-12597120y_{5n+6}+428302080y_{5n+5}\end{bmatrix}$

xiv. $\frac{1}{(12312)^3}\begin{bmatrix}3638048256y_{5n+7}-4696720298496y_{5n+5}\\+5(24y_{n+3}-30984y_{n+1})^3\\-18190241280y_{n+3}+23483601492480y_{n+1}\end{bmatrix}$

xv. $\frac{1}{(324)^3}\begin{bmatrix}85660416y_{5n+7}-325276384y_{5n+6}\\+5(816y_{n+3}-30984y_{n+2})^3\\-428302080y_{n+3}+16262881920y_{n+2}\end{bmatrix}$

2.6 Remarkable Observation

I. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in Table: 2.6.1 below.

Table : 2.6.1 HYPERBOLAS

| S.NO | HYPERBOLAS | $(X_n, Y_n)$ |
|---|---|---|
| 1. | $10X_n^2 - Y_n^2 = 26244000$ | $\begin{bmatrix}(1290x_{n+1}-30x_{n+2}),\\(120x_{n+2}-4080x_{n+1})\end{bmatrix}$ |

| | | |
|---|---|---|
| 2. | $10X_n^2 - Y_n^2 = 37896336000$ | $\begin{bmatrix}(48990x_{n+1} - 30x_{n+3}),\\(120x_{n+3} - 154920x_{n+1})\end{bmatrix}$ |
| 3. | $10X_n^2 - Y_n^2 = 29160$ | $\begin{bmatrix}(24x_{n+1} - 30y_{n+1}),\\(120y_{n+1} - 60x_{n+1})\end{bmatrix}$ |
| 4. | $10X_n^2 - Y_n^2 = 10526760$ | $\begin{bmatrix}(816x_{n+1} - 30y_{n+2}),\\(120y_{n+2} - 2580x_{n+1})\end{bmatrix}$ |
| 5. | $10X_n^2 - Y_n^2 = 15158563560$ | $\begin{bmatrix}(30984x_{n+1} - 30y_{n+3}),\\(120y_{n+3} - 97980x_{n+1})\end{bmatrix}$ |
| 7. | $10X_n^2 - Y_n^2 = 10526760$ | $\begin{bmatrix}(24x_{n+2} - 1290y_{n+1}),\\(4080y_{n+1} - 60y_{n+2})\end{bmatrix}$ |
| 8. | $10X_n^2 - Y_n^2 = 29160$ | $\begin{bmatrix}(816x_{n+2} - 1290y_{n+2}),\\(4080y_{n+2} - 2580x_{n+2})\end{bmatrix}$ |
| 9. | $10X_n^2 - Y_n^2 = 10526760$ | $\begin{bmatrix}(30984x_{n+2} - 1290y_{n+3}),\\(4080y_{n+3} - 97980x_{n+2})\end{bmatrix}$ |
| 10 | $10X_n^2 - Y_n^2 = 15158563560$ | $\begin{bmatrix}(24x_{n+3} - 48990y_{n+1}),\\(154920y_{n+1} - 60x_{n+3})\end{bmatrix}$ |
| 11. | $10X_n^2 - Y_n^2 = 10526760$ | $\begin{bmatrix}(816x_{n+3} - 48990y_{n+2}),\\(154920y_{n+2} - 2580x_{n+3})\end{bmatrix}$ |
| 12. | $10X_n^2 - Y_n^2 = 29160$ | $\begin{bmatrix}(30984x_{n+3} - 4899y_{n+3}),\\(154920y_{n+3} - 97980x_{n+3})\end{bmatrix}$ |
| 13. | $10X_n^2 - Y_n^2 = 4199040$ | $\begin{bmatrix}(24y_{n+2} - 816y_{n+1}),\\(2580y_{n+1} - 60y_{n+2})\end{bmatrix}$ |
| 14. | $10X_n^2 - Y_n^2 = 6063413760$ | $\begin{bmatrix}(24y_{n+3} - 30984y_{n+1}),\\(97980y_{n+1} - 60y_{n+3})\end{bmatrix}$ |
| 15. | $10X_n^2 - Y_n^2 = 4199040$ | $\begin{bmatrix}(816y_{n+3} - 30984y_{n+2}),\\(97980y_{n+2} - 2580y_{n+3})\end{bmatrix}$ |

II. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in Table: 2.6.2 below:

Table: 2.6.2 PARABOLAS

| S. NO | PARABOLAS | $(X_n, Y_n)$ |
|---|---|---|
| 1. | $8100X_n - Y_n^2 = 26244000$ | $\begin{bmatrix}(1290x_{2n+2} - 30x_{2n+3} + 1620),\\(120x_{n+2} - 4080x_{n+1})\end{bmatrix}$ |
| 2. | $307800X_n - Y_n^2 = 37896336000$ | $\begin{bmatrix}(48990x_{2n+2} - 30x_{2n+4} + 61560),\\(120x_{n+3} - 154920x_{n+1})\end{bmatrix}$ |
| 3. | $270X_n - Y_n^2 = 29160$ | $\begin{bmatrix}(24x_{2n+2} - 30y_{2n+2} + 54),\\(120y_{n+1} - 60x_{n+1})\end{bmatrix}$ |
| 4. | $5130X_n - Y_n^2 = 10526760$ | $\begin{bmatrix}(816x_{2n+2} - 30y_{2n+3} + 1026),\\(120y_{n+2} - 2580x_{n+1})\end{bmatrix}$ |
| 5. | $194670X_n - Y_n^2 = 15158563560$ | $\begin{bmatrix}(30984x_{2n+2} - 30y_{2n+4} + 38934),\\(120y_{n+2} - 97980x_{n+1})\end{bmatrix}$ |
| 6. | $8100X_n - Y_n^2 = 26244000$ | $\begin{bmatrix}(48990x_{2n+3} - 1290x_{2n+4} + 1620),\\(4080x_{n+3} - 154920x_{n+2})\end{bmatrix}$ |
| 7. | $5130X_n - Y_n^2 = 10526760$ | $\begin{bmatrix}(24x_{2n+3} - 1290y_{2n+2} + 1026),\\(4080y_{n+1} - 60y_{n+2})\end{bmatrix}$ |
| 8. | $270X_n - Y_n^2 = 29160$ | $\begin{bmatrix}(816x_{2n+3} - 1290y_{2n+3} + 54),\\(4080y_{n+2} - 2580x_{n+2})\end{bmatrix}$ |
| 9. | $5130X_n - Y_n^2 = 10526760$ | $\begin{bmatrix}(30984x_{2n+3} - 1290y_{2n+4} + 1026),\\(4080y_{n+3} - 97980x_{n+2})\end{bmatrix}$ |
| 10. | $194670X_n - Y_n^2 = 15158563560$ | $\begin{bmatrix}(24x_{2n+4} - 48990y_{2n+2} + 38934),\\(154920y_{n+1} - 60x_{n+3})\end{bmatrix}$ |
| 11. | $5130X_n - Y_n^2 = 10526760$ | $\begin{bmatrix}(816x_{2n+4} - 48990y_{2n+3} + 1026),\\(154920y_{n+2} - 2580x_{n+3})\end{bmatrix}$ |
| 12. | $270X_n - Y_n^2 = 29160$ | $\begin{bmatrix}(30984x_{2n+4} - 4899y_{2n+4} + 54),\\(154920y_{n+3} - 97980x_{n+3})\end{bmatrix}$ |

| | | |
|---|---|---|
| 13. | $3240X_n - Y_n^2 = 4199040$ | $\begin{bmatrix}(24y_{2n+3} - 816y_{2n+2} + 648), \\ (2580y_{n+1} - 60y_{n+2})\end{bmatrix}$ |
| 14. | $123120X_n - Y_n^2 = 6063413760$ | $\begin{bmatrix}(24y_{2n+4} - 30984y_{2n+2} + 24624), \\ (97980y_{n+1} - 60y_{n+3})\end{bmatrix}$ |
| 15. | $3240X_n - Y_n^2 = 4199040$ | $\begin{bmatrix}(816y_{2n+4} - 30984y_{2n+3} + 648), \\ (97980y_{n+2} - 2580y_{n+3})\end{bmatrix}$ |

2.7 Generation of Pythagorean triangle

2.7.1 Let p, q be the non-zero distinct integers such that

$$p = x_{n+1} + y_{n+1},\ q = y_{n+1}$$

Note that $p > q > 0$ treat p, q as the generators of Pythagorean triangle $T(X,Y,Z)$ where

$$X = 2pq\ ,\ Y = p^2 - q^2,\ Z = p^2 + q^2,\ p > q > 0$$

Let A, P represents the area and perimeter of Pythagorean triangle. Then the following results are observed.

(i) $5X - Y - 4Z = 27$

(ii) $\frac{2A}{P} = x_{n+1} * y_{n+1}$

(iii) $X + Y - \frac{4A}{P}$ is written as the sum of two squares.

(iv) $3\left(X - \frac{4A}{P}\right)$ is a Nasty number.

(v) $3(Z - Y)$ is a Nasty number.

(vi) $Z + X$ is a perfect square

2.7.2 Let p, q be the non-zero distinct integers such that

$$p = x_{n+1} + y_{n+1},\ q = y_{n+1}$$

Note that $p > q > 0$ treat p, q as the generators of Pythagorean triangle $T(X,Y,Z)$ where

$$X = 2pq,\ Y = p^2 - q^2,\ Z = p^2 + q^2,\ p > q > 0$$

Let A, P represents the area and perimeter of Pythagorean triangle. In this case, the corresponding Pythagorean triangle satisfies the relation $5Y - 4X - Z = 54$

## 3. CONCLUSION

In this paper, we have presented infinitely many integer solutions for the Diophantine equations represented by hyperbola is given by $2x^2 - 5y^2 = 27$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of equations and determine their solutions among the suitable properties.

## REFERENCES

[1] L.E. Dickson, "History of Theory of Numbers and Diophantine Analysis", Dover publications, Vol.2, New York, 2005.

[2] L.J. Mordell, " Diophantine Equations", Academic press, New York, 1970.

[3] R.D. Carmichael, "The Theory of Numbers and Diophantine analysis", Dover publications, New York , 1959.

[4] M.A. Gopalan, R. Anbuselvi, " Integral solutions of $4a^2 - (a-1)x^2 = 3a + 1$ ", Acta Ciencia Indica, XXXIV , 1, 2008, pp. 291-295.

[5] Gopalan, et al, "Integral points on the hyperbola $(a+2)x^2 - ay^2 = 4a(k-1) + 2k^2, a, k > 0$ ", Indian Journal of Science, 1(2), 2012, pp. 125-126.

[6] M.A. Gopalan, S. Devibala and S. Vidhyalakshmi, "Integral points on the Hyperbola $2X^2 - 3Y^2 = 5$ ", American Journal of Applied Mathematics and Mathematical Science, 1(1), 2014, pp. 1-4.

[7] S. Vidhyalakshmi, et al, "Observations on the hyperbola $ax^2 - (a+1)y2 = 3a - 1$ ", Discovery, 4(10), 2013 , pp. 22-24.

[8] K. Meena, M.A. Gopalan and S. Nandhini, "On the binary quadratic Diophantine equation $y^2 = 68x^2 + 13$ ", International Journal of Advanced Education and Research, 2(1), 2017, pp. 59-63.

[9] K. Meena, S. Vidhyalakshmi and R. Sobana Devi, "On the binary quadratic equation $y^2 = 7x^2 + 32$ ", International Journal of Advanced Science and Research, 2(1), 2017, pp. 18-22.

[10] K. Meena, M.A. Gopalan and S. Hemalatha, "On the hyperbola $y^2 = 8x^2 + 16$ ", National Journal of Multidisiplinary Research and Development,2(1), 2017, pp. 1-5.

[11] M.A. Gopalan, K.K. Viswanathan and G. Ramya, "On the positive Pell equation $y^2 = 12x^2 + 13$ ", International Journal of Advanced Education and Research, 2(1), 2017, pp. 4-8.

[12] K. Meena, M.A. Gopalan and S. Sivaranjan, "On the Positive Pell equation $y^2 = 102x^2 + 33$ ", International Journal of Advanced Education and Research, 2(1) , 2017, pp. 91-96.

[13] K. Meena, S. Vidhyalakshmi and N. Bhuvaneswari, "On the binary quadratic Diophantine equation $y^2 = 10x^2 + 24$ ", Intenational Journal of Multidisciplinary Education and Research, 2(1), 2017, pp. 34-39.

[14] M.A. Gopalan, V. Geetha, "Observations on some special pellian equations", Cayely J.Math, 2(2), 2013, pp. 109-118.

[15] M.A. Gopalan, S. Vidhyalakshmi and A. Kavitha, " On the integer solutions of binary quadratic equation $x^2 = 4(k^2 + 1)y^2 + 4^t, k, t \geq 0$ ", BOMSR, 2(1) , 2014, pp. 42-46.

[16] Sharadha Kumar, M.A. Gopalan, "On the hyperbola $2x^2 - 3y^2 = 23$ " , Journal of Mathematics and Informatics, Vol. 1, 2017, pp. 1-9.